

*If a straight line be cut at random and both of the segments be cut similarly to it, the greater segment of the lesser segment is equal to the lesser segment of the greater segment.*



For let the straight line  $AB$  be cut at random at  $C$ , and let its segments  $AC$ ,  $CB$  be cut similarly to  $AB$  at points  $D$ ,  $E$  respectively; [VI. 10]

I say that  $DC$  is equal to  $CE$ .

For, since  $AC$  has been cut similarly to the straight line  $AB$ ,

and  $CB$  has also been cut similarly to the straight line  $AB$ ,

therefore, proportionally, as  $AD$  is to  $DC$ , so is  $CE$  to  $EB$ ; [V. 11]

therefore also, *componendo*, as  $AC$  is to  $DC$ , so is  $CB$  to  $EB$ ; [V. 18]

and, alternately, as  $AC$  is to  $CB$ , so is  $DC$  to  $EB$ . [V. 16]

And, since  $CB$  has been cut similarly to the straight line  $AB$ ,

therefore, proportionately, as  $AC$  is to  $CB$ , so is  $CE$  to  $EB$ ;

therefore also, as  $DC$  is to  $EB$ , so is  $CE$  to  $EB$ . [V. 11]

But magnitudes which have the same ratio to the same are equal to one another;

therefore  $DC$  is equal to  $CE$ . [V. 9]

Therefore etc.

Q.E.D.